

CONVERGENCE ISSUES IN NUCLEON-NUCLEON EFFECTIVE FIELD THEORY

JAMES V. STEELE

*Department of Physics, The Ohio State University
Columbus, OH 43210-1106, USA
E-mail: jsteele@mps.ohio-state.edu*

The origin of the breakdown scale in an effective field theory treatment of nuclear forces is investigated. Different organizational schemes used in the nonperturbative calculation of nucleon-nucleon scattering seem to break down for different physical reasons. It is argued that the same physics is actually responsible, namely strong two-pion exchange as also observed in the pion scalar form factor. The prospect of extending the applicability of the effective field theory beyond this breakdown point is discussed.

1 Introduction

Strong interactions at low energies can be described by an effective field theory (EFT) of pions and nucleons with the nucleons treated nonrelativistically.¹ Chiral symmetry prescribes how pions interact. All other interactions are parameterized by the most general local Lagrangian consistent with the symmetries of quantum chromodynamics (QCD).

The coefficients of these general terms can either be predicted directly from QCD, which may be possible in future lattice calculations, or fit from existing data. I will focus on nucleon-nucleon (NN) scattering in the following. The data here is quite good and should lead to an excellent determination of these constants. However, since nuclear forces produce bound states, a nonperturbative treatment of the EFT is required, which adds a new twist to this procedure.¹

Any EFT will break down at a scale associated with the underlying physics not taken into account in the original Lagrangian. For nuclear EFTs, an optimist may believe this will occur when the center of mass momentum of the nucleons excite either the exchange of the next heaviest meson after the pion, $p \sim m_\rho$, or the nucleon- Δ mass difference $p = \sqrt{M(M_\Delta - M)} \sim 525$ MeV. In practice, the breakdown scale seems to be much lower, $p \sim 300$ MeV. This could be due to anything from quark substructure to subtleties in the fitting.

Furthermore, two different organizational schemes for calculations in the EFT seem to imply different physical reasons for the breakdown. This would be unfortunate, since alternative ways to organize the corrections should have no effect on the physics. By using some models, I will show what actually does

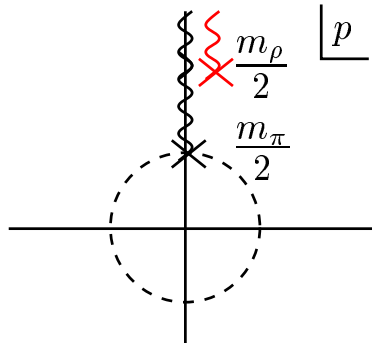


Figure 1: The analytic structure of an observable for the two-Yukawa model of Eq. (1) in the complex momentum plane.

set this scale and then use this to make connections to the real world.

Extending the applicability of the EFT to the scale associated with the Δ seems like a small achievement, but could make calculations of nuclear matter converge much faster. In that case, the momentum of interest is on the order of the Fermi momentum at saturation $p_F \sim 280$ MeV, which currently is too close to the breakdown scale for an EFT analysis to be useful.

2 Removing vs. Weakening the Pion

Assume, for now, that interactions between two nucleons are exactly given by potential exchange of both a long- and short-ranged meson with masses $m_\pi = 140$ MeV and $m_\rho = 770$ MeV respectively,

$$V_{\text{exact}}(r) = -\alpha_\pi \frac{e^{-m_\pi r}}{r} - \alpha_\rho \frac{e^{-m_\rho r}}{r}. \quad (1)$$

The pion coupling is taken to be $\alpha_\pi = g_A^2 m_\pi^2 / (16\pi f_\pi^2) \simeq 0.075$, as in the real world, and the rho coupling $\alpha_\rho = 1.05$ is tuned to give a large scattering length $a = -23.4$ fm, as observed in data for the 1S_0 partial wave of NN scattering. The following discussion does not depend on the value of the potential at the origin, so I will take V_{exact} to be zero there for simplicity.

This potential generates observables with analytic structure as shown in Fig. 1, specified by a cut at both $m_\pi/2$ and $m_\rho/2$. The existence of these cuts indicates that a low-momentum expansion of an observable will not converge

for momenta above $m_\pi/2$. An EFT can only go beyond this point if it either exactly reproduces or sufficiently approximates the analytic structure of the pion.

This can be illustrated by focusing on the most general Lagrangian without pions (I will restrict myself to the 1S_0 channel below)

$$\mathcal{L}_{\text{EFT}} = N^\dagger \left[i\partial_t + \frac{\nabla^2}{2M} \right] N - C_0(N^\dagger N)^2 + \frac{1}{2}C_2 [(N^\dagger \nabla N)^2 + h.c.] + \dots, \quad (2)$$

which corresponds to a nonrelativistic potential between two nucleons of

$$V_{\text{EFT}}(p, p') = C_0 + \frac{1}{2}C_2 (p^2 + p'^2) + \dots, \quad (3)$$

where p and p' are the center of mass momentum of the incoming and outgoing nucleons. Since these interactions depend on powers of momentum, a naive calculation of the NN scattering phase shifts from \mathcal{L}_{eff} leads to ultraviolet divergences. This just reflects our ignorance of the high-energy behavior of this theory.

Two possible procedures to regulate these divergences are: i) adding a cutoff to all integrals to eliminate or suppress the high momentum states and ii) using dimensional regularization, which amounts to throwing out any integrals that diverge like powers of momentum. The second procedure may seem a bit counterintuitive at first, but is simpler to implement since it does not break any symmetries of the underlying theory and therefore does not require additional counterterms. Perturbative calculations in chiral perturbation theory have confirmed the equivalence of these regularization procedures.

Nonperturbative calculations, on the other hand, show that a naive implementation of dimensional regularization is incomplete. A generalization to account for integrals that linearly diverge is required. This regularization is called power divergence subtraction² (PDS).

After regulating the theory, the EFT can be organized to give predictions order-by-order in powers of an expansion parameter. Since the low-energy constants C_{2n} are not determined by the symmetries of QCD, they must be fit to data. In the process, they will typically pick up the next scale Λ of physics not explicitly accounted for in the EFT, leading to $C_{2n}p^{2n} \sim (4\pi/\Lambda^2)(p/\Lambda)^{2n}$. Therefore interactions with more derivatives are suppressed by additional powers of p/Λ . Power counting in this parameter allows Eq. (2) to be truncated to the first few terms, with more accurate results obtained by keeping more terms in the momentum expansion.

There are two prevalent ways of implementing power counting in nonperturbative effective field theories. The first is to count powers of Λ in the potential V_{EFT} and then solve the Lippmann-Schwinger equation. Such a procedure

sums an infinite number of Feynman diagrams to generate the nonperturbative amplitude \mathcal{A} , related to the phase shift δ by

$$\mathcal{A} = \frac{4\pi/M}{p \cot \delta - ip}. \quad (4)$$

This method of power counting is called Λ -counting³ and is in the spirit of the Hamiltonian theory of Wilson, first applied to nuclear EFTs by Weinberg.⁴ An alternative is to expand \mathcal{A} in powers of $Q \equiv (p, m_\pi, 1/a)$, motivated by chiral perturbation theory, which expands in p and m_π . The nonperturbative nature of the interactions identifies $1/a$ as a small parameter as well, justifying its inclusion in the expansion. This method of power counting is called Q -counting.^{2,5}

For a theory without pions, such as Eq. (2), the underlying scale Λ is just m_π . Both power counting procedures work as long as momenta are less than this scale, reproducing the effective range expansion

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2}p^2 \sum_{n=0}^{\infty} r_n \frac{p^{2n}}{\Lambda^{2n}}. \quad (5)$$

The effective ranges, which are determined from NN scattering data,⁶ also exhibit this scale $r_n \sim 1/\Lambda \sim 1/m_\pi$. To extend the EFT to higher momenta, pions need to be explicitly added to the effective Lagrangian. This hopefully will bring Λ up to the QCD scale of m_ρ or possibly even 1 GeV.

The inclusion of pions nonperturbatively is different in the two power counting procedures. Every iteration of one-pion exchange contributes to the pion cut in Fig. 1. In general, n iterations are proportional to^a

$$n\text{-Pion Exchange} \propto \left[\frac{m_\pi}{\Lambda_{\text{NN}}} \frac{m_\pi^2}{4p^2} \ln \left(1 + \frac{4p^2}{m_\pi^2} \right) \right]^n, \quad (6)$$

where $\Lambda_{\text{NN}} \simeq 300$ MeV is just another way of expressing the strength of the interaction via the relation $\alpha_\pi = m_\pi^2/(M\Lambda_{\text{NN}})$. Λ -counting includes this long-range part of one-pion exchange fully at leading order, removing the effect of the pion cut. Q -counting pushes one-pion exchange to subleading order and iterated one-pion exchange to even higher orders. Therefore, each order in the Q -counting expansion weakens the pion cut by an additional power of $m_\pi/\Lambda_{\text{NN}}$, as seen in Eq. (6). The topic of the next section is whether this simpler implementation of power counting, also called perturbative pions, can be as successful as Λ -counting.

^aThis can be seen by studying the exact solution of an exponential potential,⁸ which has the same cut structure as a Yukawa or by looking at the analytic expressions of higher Yukawa loops, which exhibit the cut structure through Polylogarithms.⁹

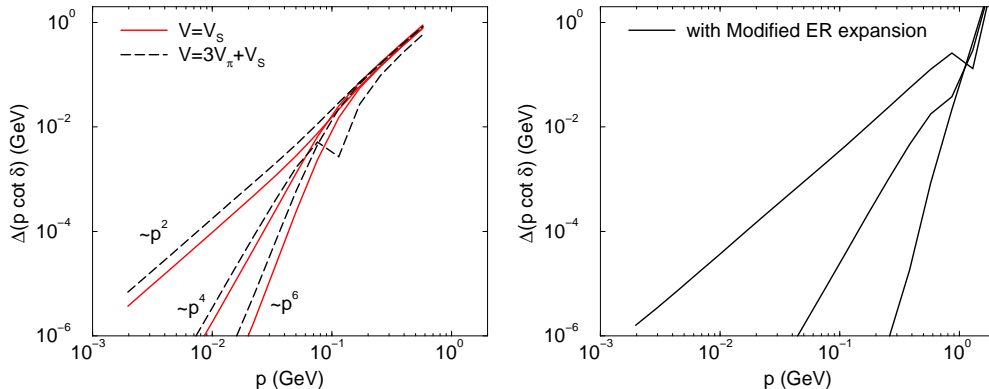


Figure 2: The error in $p \cot \delta$ for a pion coupling three times stronger than the real world. The left plot is from a normal fit to the low-energy constants both without (solid) and with (dashed) explicit pions. The right plot uses the modified effective range expansion to fit.

3 New Fitting Procedures

The NN EFT is expected to work best at low-momentum. So a natural procedure to fix the low-energy constants C_{2n} is to calculate an observable, like $p \cot \delta$, and fit near threshold. However, there are some complications originating from the nonperturbative nature of the problem. These must be cleared up before being able to make a statement about the breakdown scale.

3.1 Λ -counting and the Modified Effective Range Expansion

Most calculations that implement Λ -counting require a numerical treatment (although there are exceptions⁵). A nice way to show the systematic improvement expected of an EFT in this case is to use error plots^{3,7,8} as in Fig. 2. As more and more constants are added to the effective potential Eq. (3), the error in $p \cot \delta$ improves systematically by powers of p^2 , reflected in the improving slopes. The point at which all errors are of the same order is where the theory is expected to break down.⁷ The solid lines of the left plot of Fig. 2 show the case without explicit pions. The breakdown point, or radius of convergence, of the EFT is around $m_\pi/2$, as expected from the discussion in Section 2.

Adding pions to the short-ranged Lagrangian Eq. (2) as dictated by chiral symmetry should improve the radius of convergence.¹⁰ However, one needs to be careful when implementing this with cutoff regularization. If the pion coupling were three times stronger, for example, adding it to the Lagrangian

$$\begin{aligned}
\delta_\pi &\rightarrow \overline{\vdots} + \overline{\vdots} + \dots \\
\frac{1}{\mathcal{F}_\pi} &= \angle + \angle + \angle + \dots \\
\frac{f'_\pi(p,0)}{\mathcal{F}_\pi(p)} &= \text{fish} + \text{fish} + \dots
\end{aligned}$$

Figure 3: Symbolic definition of quantities in the modified effective range function in Eq. (7), which sums all the pion contributions.

would *not* improve the breakdown scale.⁸ This is illustrated by the dashed lines in the left plot of Fig. 2. Chiral symmetry helps in that the coupling is small, but an EFT should work regardless of the strength of the potential being added. What went wrong?

Note that the effective ranges r_n of Eq. (5), which are used to fix the values of the C_{2n} , exhibit the scale m_π . If this scale were completely removed from the fitting procedure, then the constants should take on the proper scale m_ρ . One way to do this is to construct a function similar to $p \cot \delta$ which is analytic all the way to the rho cut in Fig. 1. In proton-proton scattering analyses, such a procedure is used to remove the threshold cuts coming from photon exchange. Called the modified effective range expansion,¹¹ it can be used here to remove the effect of pion exchange by defining

$$K(p) \equiv \frac{p \cot(\delta - \delta_\pi)}{|\mathcal{F}_\pi(p)|^2} + \text{Re} \left[\frac{f'_\pi(p,0)}{\mathcal{F}_\pi(p)} \right] \equiv -\frac{1}{a} + \frac{1}{2} p^2 \sum_{n=0}^{\infty} \tilde{r}_n \frac{p^{2n}}{\tilde{\Lambda}^{2n}}. \quad (7)$$

All possible insertions of one-pion exchange are taken into account in this formula, as seen pictorially in Fig. 3. For the model potential Eq. (1), this function is analytic up to $m_\rho/2$ and has modified effective ranges $\tilde{r}_n \sim 1/\tilde{\Lambda} \sim 1/m_\rho$. Fitting the C_{2n} 's using the function $K(p)$ leads to much better results, shown in the right plot of Fig. 2. The radius of convergence is now on the order of m_ρ as expected.

Using a cutoff to regulate the theory mixes long- and short-distance physics, which is what led to the premature breakdown.⁸ The modified effective range expansion removes the long-distance physics from the observable, allowing a clean fit of the low-energy constants to the short-distance physics. In principle, this can also be achieved by the addition of more counterterms, but the modified effective range expansion is a simpler alternative. Luckily, one-pion exchange in the real world is weak enough that an improper treatment of this

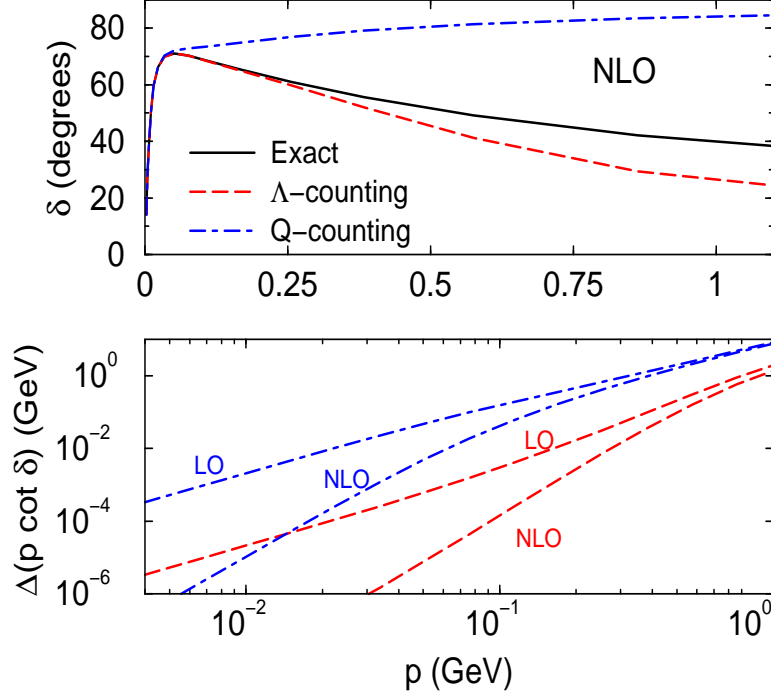


Figure 4: Comparison of the two power counting procedures for the two-Yukawa model Eq. (1) using a fit at low momentum.

malady does not make a difference.⁸ However, this might not be the case for two-pion exchange as will be discussed in Section 4.

3.2 *Q-counting and a Global Fit*

A low-momentum fit to data using Q -counting and perturbative pions also breaks down much earlier than expected.⁸ In Fig. 4, the leading order (LO) and next-to-leading order (NLO) calculations using Q -counting are compared to those using Λ -counting. The Λ -counting result has a radius of convergence around m_ρ after the low-energy constants are matched to the modified effective range expansion.

Implementing the modified effective range expansion is inconsistent with the idea of perturbative pions, and so the Q -counting result breaks down around m_π . However, it is important that the low-energy constants only pick

up the (unknown) short-distance physics, and matching to the modified effective range expansion is the most convenient way to do this. Using PDS, this would lead to the expressions¹²

$$C_0 = \frac{4\pi/M}{-\mu + 1/\tilde{a}}, \quad C_2 = \frac{MC_0^2}{4\pi} \frac{1}{2} \tilde{r}_0, \quad \dots, \quad (8)$$

which only depend on the short-distance physics.^b One important modification to this would be to match the pole of the amplitude \mathcal{A} exactly¹³

$$C_0 = \frac{4\pi/M}{-\mu + 1/a}. \quad (9)$$

Otherwise, terms attempting to correct the position of the pole would enter as double poles at higher orders messing up the power counting. This modification requires perturbative corrections to C_0 that compensate the pion contributions at each order. However, this still leaves the other C_{2n} 's dependent on the modified effective ranges \tilde{r}_n , which cannot be obtained using perturbative pions.

A method must therefore be devised to carry out the matching of Eq. (8). It is reasonable to assume that when the modified effective ranges are set to the correct values, the error of the EFT will be less than when the ranges are at some random values. This can be checked by using a model. Replacing the Yukawas in Eq. (1) by delta-shells

$$-\alpha_i \frac{e^{-m_i r}}{r} \rightarrow -g_i \delta\left(r - \frac{1}{m_i}\right), \quad (10)$$

an analytic expression for the exact answer can be obtained.¹² Expanding the exact amplitude in powers of Q , it can be compared to the blind EFT fit of the low-energy constants to data using Q -counting. The results coincide best when the modified effective ranges are varied to globally minimize the χ^2 for data in a representative momentum range like $[m_\pi, m_\rho]$.

The results are shown up to next-to-next-to-leading order (NNLO) in Fig. 5. They suggest that for Q -counting, a constrained global fit of the EFT to data to fix the low-energy constants is better than a low-momentum fit. The constraint is that the modified effective ranges, not the low-energy constants themselves, are varied to achieve the best fit.

One thing to notice about Fig. 5 is that the errors do not show the systematic improvement in slope characteristic of the earlier error plots. Although the

^bAlthough the renormalization scale μ appears in these low-energy constants, all observables are independent of it.

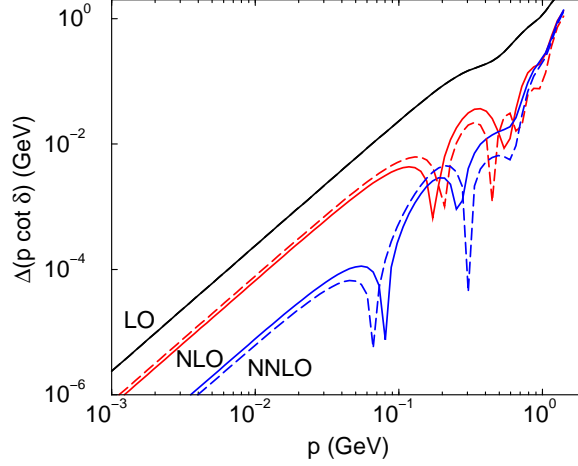


Figure 5: Q -counting expansion of the two delta-shell model. The exact expression (solid) coincides nicely with the EFT (dashed) when a global fit is minimized with respect to the modified effective ranges. The dashed line at LO is on top of the solid line.

scattering length is fit exactly to reproduce the pole in the amplitude Eq. (9), the other terms in the effective range expansion have corrections at each order in the Q -counting. A comparison of Fig. 5 with the right plot of Fig. 2 shows that accuracy is sacrificed near threshold to obtain the global fit needed for this power counting. In addition, the ability to read the EFT radius of convergence from the error plot is gone.

Another way to estimate the breakdown scale is therefore needed. The calculated amplitude \mathcal{A} starts at order Q^{-1} and has a contribution \mathcal{A}_n at each order Q^n in the expansion. Up to this point, I have extracted the Q -counting phase shift by expanding the expression²

$$\delta = \frac{1}{2i} \ln \left[1 + i \frac{Mp}{2\pi} (\mathcal{A}_{-1} + \mathcal{A}_0 + \dots) \right] \quad (11)$$

$$\rightarrow \frac{1}{2i} \ln \left(1 + i \frac{Mp}{2\pi} \mathcal{A}_{-1} \right) + \frac{Mp}{4\pi} \left(\frac{\mathcal{A}_0}{1 + i \frac{Mp}{2\pi} \mathcal{A}_{-1}} \right) + \dots, \quad (12)$$

but I could also have kept the full expression Eq. (11). These two expressions are equivalent up to terms that are higher order in the Q expansion. Those extra terms, though, are an estimate of the corrections to the actual result and depend on the radius of convergence. The breakdown for Q -counting can

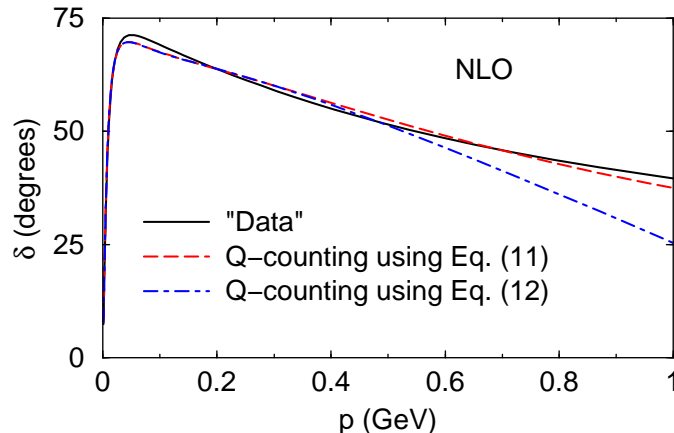


Figure 6: Determination of the Q -counting breakdown scale for the two-Yukawa model.

be identified as the point at which the two expressions Eqs. (11) and (12) diverge from each other. Doing this exercise for the two-Yukawa model results in Fig. 6. There it is easy to see the actual breakdown is somewhere around m_ρ as expected.

4 Data and the Role of Two-Pion Exchange

Now that the two power counting methods are well understood and give the proper radius of convergence for a model, it is appropriate to apply them to actual NN-scattering data. Using the Nijmegen partial wave analysis,⁶ the radius of convergence for each of the two different power counting schemes is shown in Fig. 7. The left plot shows that, for Λ -counting with one-pion exchange included as in the previous section, the breakdown is around $p \sim 300$ MeV. The right plot shows a similar scale for Q -counting with perturbative pions.

This scale is less than expected and seems to be set by physics other than the ρ or Δ . Early calculations in Q -counting predicted a breakdown at Λ_{NN} , which would coincide with the observed scale. However, recall that in the previous section I showed that Q -counting broke down around m_ρ in the two-Yukawa model. That calculation had identical content with respect to one-pion exchange, indicating that the 300 MeV comes from different physics. A closer study of Λ -counting determines that two-pion exchange should be included at

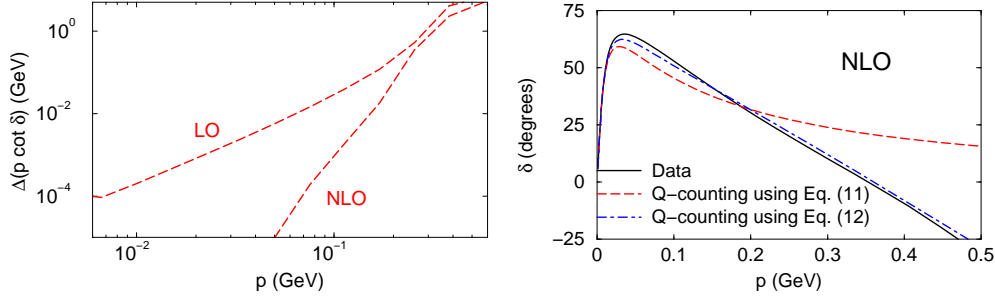


Figure 7: Results for 1S_0 NN-scattering data. The breakdown scale is determined by the left plot for Λ -counting and the right plot for Q -counting.

NLO.¹⁰

The fact that the actual data behaves so much differently from the two-Yukawa model indicates the model was not sophisticated enough. Furthermore, no reasonable choices for the parameters of this model can give as large an effective range $r_0 = 2.63$ fm as seen in the 1S_0 channel.⁸ Until now I have assumed that m_π and m_ρ are the underlying scales, but nuclear physics wisdom tells us there are actually two short-range Yukawas

$$V(r) = -\alpha_\pi \frac{e^{-m_\pi r}}{r} - \alpha_\sigma \frac{e^{-m_\sigma r}}{r} + \alpha_\rho \frac{e^{-m_\rho r}}{r}, \quad (13)$$

given by an attractive scalar (of mass between 500 and 600 MeV) and repulsive vector particle.^c Taking the short-distance couplings to be much stronger than the pion $(\alpha_\sigma, \alpha_\rho) = (7, 14.65)$, the modified effective ranges \tilde{r}_n of the data can also be reproduced.⁸ This three-Yukawa model is reminiscent of the Bonn potential,¹⁴ which is known to model the data well.

Applying the NN EFT to observables produced from Eq. (13) indeed leads to a breakdown around $m_\sigma/2 \sim 300$ MeV. I can verify the breakdown is not set by the two-pion threshold $2m_\pi$ within this model by varying the pion and scalar masses. The $\sigma(600)$ particle is a common parameterization in nuclear physics for two-pion exchange contributions. The large scalar coupling indicates that including such effects through chiral symmetry might require the modified effective range expansion. Such an analysis could improve the EFT radius of convergence.^d

^cThe vector particle is normally taken to be the ω meson, but for continuity with the earlier discussion, I will call it the ρ meson.

^dA preliminary calculation, inspired by discussions at this workshop, seems to indicate this is true.¹⁵

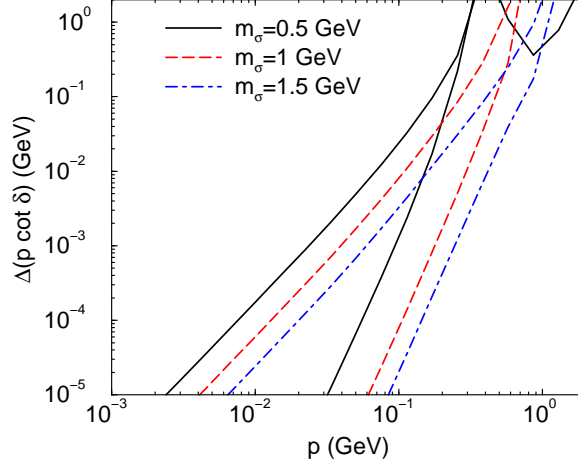


Figure 8: The three-Yukawa model Eq. (13) with different values for the scalar mass, but a , r_0 , and $m_\rho - m_\sigma$ fixed. This shows that the radius of convergence tracks with the changing m_σ .

It might seem odd that two-pion effects could be strong, since they should be weak according to chiral symmetry.³ However, there are other instances where two-pion effects have proven to be strong. The pion scalar form factor, defined by

$$\langle \pi^a(p) \pi^b(p') | \hat{m} (\bar{u}u + \bar{d}d) | 0 \rangle = \delta^{ab} \sigma_\pi \Gamma(s), \quad s = (p + p')^2, \quad (14)$$

has been shown to have poor convergence in chiral perturbation theory,¹⁶ breaking down just above threshold at $\sqrt{s} \simeq 400$ MeV. This comes about from infrared effects that require the summation of $(\ln s)^n$ terms to all orders. Carrying out this summation does lead to improved results and the generation of a “resonance”-like shape in the dispersion analysis just like a scalar resonance around 600 MeV.¹⁶ The same phenomenon could occur in NN scattering and requires further study.

5 Red Herrings

The models developed to understand the breakdown scale better can also be used to shed light on other convergence issues. One feature of the three-Yukawa model is that it can reproduce the somewhat large effective range in NN scattering, which is $r_0 \sim 2/m_\pi$. Any reasonable choices of parameters for

the two-Yukawa model can only give r_0 around $1/m_\pi$. Therefore, it is possible that the lower radius of convergence of the three-Yukawa model might have nothing to do with two-pion effects, but come from some type of correlation with the effective range r_0 instead. Choosing different parameters in Eq. (13), I can keep a , r_0 , and the mass difference $m_\rho - m_\sigma$ fixed while changing m_σ from 500 MeV to 1500 MeV. The results are shown in Fig. 8, confirming that the radius of convergence is set by $m_\sigma/2$ alone.

Recently, the convergence of Q -counting has been challenged within the context of a low-energy theorem.⁵ The point can be made by defining the observable

$$S(p^2) \equiv p \cot \delta - \left(-\frac{1}{a} + \frac{1}{2} r_0 p^2 \right), \quad (15)$$

and taking $p = |1/a|$. Then Q -counting has two types of corrections:

$$\frac{1}{a\Lambda_{\text{NN}}} \ll 1, \quad \frac{m_\pi}{\Lambda_{\text{NN}}} \sim \frac{1}{2}. \quad (16)$$

It was shown that the Q -counting prediction for $S(p^2)$ at NLO is very far from the data.

This can also be seen in a model. For an analytic understanding, I will use the two delta-shell potential of Section 3. Expanding in Q ,

$$S\left(\frac{1}{a^2}\right) = (-22.9 + 40.2 - 23.3 + 7.9 + \dots) \times 10^{-7} m_\pi, \quad (17)$$

NLO NNLO ...

one finds the NNLO term is larger than the NLO term. Only after a few more orders does the series start to converge.

Taking the exact answer, I can also expand in Q to see what is happening analytically. These expressions are messy and can be simplified by observing that even though $m_\pi a = \mathcal{O}(1)$ in Q -counting, it is actually of order 10 numerically. Expanding $S(1/a^2)$ in $(m_\pi a)^{-1}$ then should simplify the expression immensely without blurring its meaning, leading to

$$S\left(\frac{1}{a^2}\right) = \frac{m_\pi}{(m_\pi a)^4} \left[-\frac{m_\pi}{3\Lambda_{\text{NN}}} + \frac{m_\pi^2}{\Lambda_{\text{NN}}^2} \left(\frac{13}{15} + \frac{8\Lambda_{\text{NN}}}{9m_\rho} \right) + \dots \right] \quad (18)$$

$$+ \frac{m_\pi}{(m_\pi a)^5} \left[\frac{4m_\pi}{15\Lambda_{\text{NN}}} - \frac{m_\pi^2}{\Lambda_{\text{NN}}^2} \left(\frac{68}{45} + \frac{4\Lambda_{\text{NN}}}{9m_\rho} \right) + \dots \right] + \dots \quad (19)$$

Terms in the second bracket and higher are suppressed by $(m_\pi a)^{-1}$ and can be neglected. Focusing on the first bracket of terms, it is clear that the expansion

is in $m_\pi/\Lambda_{\text{NN}}$ as expected for Q -counting. However, the leading term (which happens to be NLO for this particular observable) has a numerical factor of $\frac{1}{3}$, which suppresses it down to the order of the NNLO term. Therefore, the Q expansion of $S(p^2)$ works, but is slowly converging due to some inconvenient numerical factors as well as the size of the expansion parameter, Eq. (16).

6 Summary

Nonperturbative effective field theories are subtle to implement. Techniques used successfully in chiral perturbation theory, such as naive dimensional regularization and low-momentum fitting, do not produce the expected results in nonperturbative calculations. After a careful study using solvable models, new methods have been developed to deal with these complications.^{2,8,12}

Pions enter the calculation differently depending on whether power counting is done in the QCD scale (Λ -counting) or the chiral scale (Q -counting). In both cases a proper determination of the low-energy constants from the short-distance physics requires more than just a low-momentum fit. Utilizing the modified effective range expansion to remove the effect of the long-distance pions from the process of fitting, the Λ -counting radius of convergence moves beyond the pion scale.⁸ A constrained global fit minimized with respect to the modified effective ranges is required to achieve the same effect in Q -counting.¹²

Applying the new fitting procedures to 1S_0 NN scattering data results in a breakdown of about 300 MeV. Models can be used to confirm this breakdown is associated with the same physics for both power counting procedures. Using a three-Yukawa model, it was argued that including two-pion exchange will extend the EFT radius of convergence. It is not unreasonable to assume this effect is strong since a similar situation occurs in the pion scalar form factor.¹⁶

Finally, the convergence of Q -counting was investigated using an exactly solvable model. It was shown that some observables could have inconvenient numerical factors, such as occur in the low-energy theorem,⁵ which suppress the leading orders and mask the behavior of the expansion. However, analytical results show that this expansion does converge, albeit slowly.

Acknowledgments

The work presented in this talk was done in collaboration with R. J. Furnstahl and D. B. Kaplan. I would like to thank the organizers, P. Bedaque, M. J. Savage, R. Seki, and U. van Kolck, for an enjoyable and productive workshop and the Institute of Nuclear Theory at the University of Washington for its hospitality. This work was supported by the National Science Foundation under Grants No. PHY-9511923 and PHY-9258270.

References

1. *Nuclear Physics with Effective Field Theory*, ed. R. Seki, U. van Kolck, and M. J. Savage (World Scientific, 1998);
U. van Kolck, “Effective Field Theory of Nuclear Forces,” nucl-th/9902015, and references therein.
2. D. B. Kaplan, M. J. Savage, and M. B. Wise, *Phys. Lett. B* **424**, 390 (1998); *Nucl. Phys. B* **534**, 329 (1998).
3. G. P. Lepage, “How to Renormalize the Schrodinger Equation,” nucl-th/9706029;
G. P. Lepage, *this volume*.
4. S. Weinberg, *Phys. Lett. B* **251**, 288 (1990); *Nucl. Phys. B* **363**, 3 (1991).
5. T. D. Cohen and J. M. Hansen, nucl-th/9808006; nucl-th/9808038; nucl-th/9901065;
T. D. Cohen, *this volume*.
6. V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, *Phys. Rev. C* **48**, 792 (1993).
7. J. V. Steele and R. J. Furnstahl, *Nucl. Phys. A* **637**, 46 (1998).
8. J. V. Steele and R. J. Furnstahl, *Nucl. Phys. A* **645**, 439 (1999).
9. M. Binger, “Multiloop Diagrams in Nonrelativistic Field Theories and the Deuteron Quadrupole Moment,” nucl-th/9901012;
G. Rupak, *this volume*;
T. Mehen, *this volume*.
10. U. van Kolck, Ph.D. thesis, U. Texas, Aug. 1993;
C. Ordonez, L. Ray, and U. van Kolck, *Phys. Rev. Lett.* **72**, 1982 (1994); *Phys. Rev. C* **53**, 2086 (1996).
11. H. van Haeringen and L. P. Kok, *Phys. Rev. A* **26**, 1218 (1982);
A. M. Badalyan, L. P. Kok, M. I. Polikarpov, and Y. A. Simonov, *Phys. Rep.* **82**, 31 (1982).
12. D. B. Kaplan and J. V. Steele, in preparation;
D. B. Kaplan, *this volume*.
13. T. Mehen and I. W. Stewart, *Phys. Lett. B* **445**, 378 (1999).
14. R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).
15. J. V. Steele and R. J. Furnstahl, in preparation.
16. J. Gasser and U.-G. Meissner, *Nucl. Phys. B* **357**, 90 (1991).